Developing Young Children's Mathematical Power

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Enrichment for mathematically gifted students in the elementary school needs to extend beyond puzzles or busywork and support the development of mathematical power through a differentiated curriculum. This article describes a series of enrichment experiences that were designed to develop young gifted children's understanding of large numbers, which was central to their investigation of space travel. Although large numbers are not traditionally included in the mathematics curriculum for young children, they responded enthusiastically to the enrichment experiences. These experiences provided the children with an opportunity to understand the large numbers they encountered in science resource material and, importantly, to develop their mathematical power.

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Large numbers are a source of fascination for many children and mathematicians alike. However, unlike mathematicians, most young children have a limited understanding of large numbers and use place value terms indiscriminately to express the enormity of a quantity or measure — "There were thousands of people at the party"; "The house cost millions of dollars". Traditionally, large numbers have not been part of the mathematics curriculum in the early school years. However, a lack of understanding of large numbers can be problematic for young gifted children because large numbers are an integral part of topics that are of interest to them, such as space travel.

In our work with enrichment classes of five-to-eight year-olds, we found that children were hampered in their investigation of space travel when large numbers were encountered in resource material. Eventhough these young gifted children were more mathematically competent than their chronological peers in their regular classrooms, they had difficulty in appreciating the number of people who watched the first moon landing; the size of the space mission team; the cost of a space mission; and the distances from the earth to the moon, the planets, and the stars. When children lack an understanding of large numbers, they are unable to reason effectively with the information given. For example, one child reasoned that the moon must be closer than a city because "You can see the moon at night but you can't see Sydney". Thus, for children to take advantage of the information in the space resource material, there was a need to develop their number sense with large numbers, that is, their multidigit number sense.

Multidigit number sense refers to:

> Children's understanding of and flexibility in using multiunit numbers … [this] understanding should also include intuitive feelings for numbers and their uses as well as the ability to make judgements about the reasonableness of multidigit numbers in diverse problem situations. (Jones, Thornton, & Putt, 1994, p. 118)

Although multidigit number sense is complex (Jones et al, 1994), it was necessary to develop a series of meaningful activities that would enable young gifted children to make sense of large numbers in context. Enrichment classes provide the opportunity for gifted students to engage in tasks that are beyond the scope of the regular curriculum (Lupkowski-Shoplik & Assouline, 1994). The following enrichment activities were designed to help a class of twenty young gifted children to:

1. Read (i.e., label) large numbers in symbolic form;
2. Develop referents for large numbers and understand their relative magnitude (National Council of Teachers of Mathematics, 1998); and
3. Understand large numbers that represent quantity (e.g., size of a space mission team), distance (e.g., distance to the moon) and money (e.g., cost of a space mission).

READING LARGE NUMBERS

In the first activity, children were introduced to the pattern in reading large numbers. Numbers of increasing magnitude were displayed for the children. We began with the one's period, progressed to the thousand's period, and finally, displayed the million's period. The name of each period was added to facilitate children's reading, as shown in Figure 1.

Insert Figure 1

The children enjoyed reading these numbers and were keen to read further in the millions period. Their responses indicated that they had developed the basic skill of reading large numbers: "I learnt that if you try, you can not only count to, but also read large numbers" (Karen, eight years). Furthermore, even the youngest children demonstrated a primitive knowledge of the structure of our number system as can be seen in six-year-old Mark's recording of this activity (Refer to Figure 2).

Insert Figure 2

Following this, the children progressed to three problem-based activities designed to develop the second and third aims, which focused on children's understanding of large numbers in relation to quantity, distance, and money. The children rotated through these activities in small groups.

UNDERSTANDING LARGE NUMBERS

Activity 1: Estimating a Large Quantity

How many peas did Frank knock off his plate?

Insert Figure 3

Clement, Rod. "Counting on Frank." Sydney: Angus & Robertson, 1990.

The book "Counting on Frank" (Clement, 1990) was read to the group and the children were challenged to determine how many peas Frank knocked off his plate (Refer to Figure 3). Initially, the children gave answers such as "a hundred", "a thousand" and "a million" but were unable to agree or explain their estimates. When asked how to find out which answer was correct, their only suggestion was to "count the peas". Other children rejected this suggestion because "It would take too long" and "You can't see all the peas". To provide a basis for solving this problem, the children explored numbers to a thousand and then to one million.

Exploring Numbers to a Thousand

Referents for 1, 10, 100 and 1000 were developed using colored sprinkles (confectionery decoration) on buttered bread that was cut into four pieces. The children added sprinkles as follows:

1. 1 sprinkle was placed on the first piece of bread.
2. 10 sprinkles were counted out onto the next piece.
3. Approximately 100 sprinkles were placed on the next piece. Instead of counting, groups of ten were estimated until 100 was reached.
4. Approximately 1000 sprinkles were placed on the final piece. Groups of one hundred were estimated until 1000 was reached.

The sprinkles activity provided a meaningful referent for children's understanding of the relative magnitude of numbers to a thousand, as shown in eight-year-old David's recording (See Figure 4).

Insert Figure 4

Some children extrapolated beyond the physical referent, making comments such as, “I learnt that you probably can’t fit one million sprinkles on one piece of bread”. (Helen, eight years)

We then revisited the original problem about the number of peas Frank knocked off his plate. Although the children were confident that there were more than 1000 peas, they were unable to suggest an approximate number. We then put the problem aside and explored numbers to a million.

Exploring Numbers to a Million

Multi-base arithmetic blocks [MAB] were used to provide a physical referent for one million. The children first explored the area of the meter square using MAB ones, tens and hundreds. They then investigated the capacity of a meter cube using MAB hundreds and thousands. Eight-year-old Aidan explained this process: “We got 12 one-meter rulers, and hundreds of MAB blocks and thousands of MAB blocks. We multiplied and knew how many MAB blocks [would fill the meter cube]”.

With some appreciation of numbers to a million, the children were now able to deal with Frank’s problem. They reasoned that, because the size of a single MAB was comparable to a pea, Frank had knocked off between two million and five million peas. It was clear that these large number experiences had informed the children’s reasoning in working this problem. For example, eight-year-old Helen explained “There would be about two million (peas) because the peas are higher than the meter cube and you can’t see all the peas in the picture”.

Activity 2: Appreciating Monetary Value

What sized container would be needed to carry a million dollars?

In preparation for solving the focus problem above, the children completed two tasks: Money Posters and Monopoly Money. These tasks are described prior to addressing the children’s responses to the focus problem.

Money Posters

Posters were labelled with the amounts $1, $10, $100, $1000, $10 000, $100 000 and $1 000 000. The children identified items in magazine advertisements that approximately cost each of these amounts. They then completed the posters by glueing items under the corresponding amounts. It was evident that this activity raised children’s awareness of the monetary value of expensive items, as can be seen in seven-year-old Sondra's comment. "I found out that money could go very large and that you could buy things with a thousand dollars".

Insert Figure 5

Monopoly Money

The children were then given the open-ended task of calculating how much money was in a Monopoly game. Although calculators were provided, the children did not necessarily use them efficiently. For example, one pair of children tediously entered the value of each note in turn, without considering different ways of grouping the notes. In contrast, some children who chose to work manually displayed increasing sophistication in their calculations. This can be seen in Karen's work (See Figure 6).

Insert Figure 6

After the children had completed these preparatory tasks, they then tackled the focus problem of determining the container size needed to hold a million dollars. The children used the Monopoly money as a referent for working this problem. No containers were provided, rather, the children were encouraged to model different container sizes with their hands. Through discussion, the children realised that there was more than one answer to the problem.

They reasoned that the size of the container was dependent on the denomination of the notes that were used to make one million dollars. Some children commented that a larger sized container would be required if notes of low value were used and vice versa. In other words, the size of the container would be inversely proportional to the denomination of the note that was used. In their search for the smallest container possible, the children's discussion extended to the highest denomination of notes available in other countries.

Activity 3: Stars and Light Years

How far away are the brightest stars?

The purpose of this activity was to develop children's understanding of large distances within the context of space travel. Prior to exploring this problem, the book "How Much is a Million?" (Schwartz, 1985) was read and discussed. The children then made ten paper stars, which were labelled with the names of the ten brightest stars (in the Southern Hemisphere), their brightness, and their distance from the earth. The stars were fastened onto upturned paper cups for ease of mobility. The children initially ordered the stars by brightness beginning with the brightest star.

Next, consideration was given to the stars' distances from the earth. After the children had discussed the notion of measuring stellar distances in light years, they reordered the stars from the closest to the most distant. The children spontaneously debated whether there was a correspondence between the brightness of a star and its distance from earth.

To represent the stars' relative distances from the earth in light years, a timeline was drawn and marked in 100s from 0 to 1000. The children positioned each star at the correct number of light years from the earth (See Figure 7). They then discussed the idea that when we see a star today, the light from that star was actually emitted a number of years ago. For example, Alpha Centuri is about four light years away and so what is seen is the star as it was four years ago. The children were fascinated with this idea and began working out the year when light was emitted from particular stars. They were encouraged to relate these years to significant events, such as the year of their birth and the beginning of the century. The discussion of stars extended to the birth and death of stars, and the fact that some of the stars we see may no longer exist.

Insert Figure 7

This activity enabled children to make links between their mathematical understanding and their scientific knowledge. Eight-year-old Adam's comment illustrates this: "I learned how far away and how bright some stars are. All stars are heaps of light years away except for the sun. The sun is the closest star to the earth. It is a medium-sized star".

CONCLUSION

The children's responses throughout the activities suggests aspects of multidigit number sense that need consideration in teaching mathematically gifted elementary students. For example, some children were unaware of the existence of large numbers: "I never knew that there was such a thing as one hundred thousand" (Shaun, eight years). Others, such as six-year old Sandy, were amazed at the number of digits needed to represent large numbers symbolically (e.g., 1 000 000), in contrast to their verbal name (one million). The visual impact of constructing a million (meter cube) was evident in the comments of other children, such as Bryan (six years) who recorded "I learnt that numbers could take up so much space".

The activities presented here fulfilled our aims of developing children's multidigit number sense to facilitate their understanding of the space travel resource material. At the same time, the children developed a fascination for large numbers, and derived enjoyment from conducting mathematical investigations. Additionally, there were opportunities for the children to develop their logico-mathematical intelligence and spatial intelligence (Gardner, 1983; Kruteskii, 1976). Most importantly, however, the children's reflection on their learning empowered their work as mathematicians. For example, Drew (eight years) commented, "I learned that if you **think**, you can figure out how to count large numbers [emphasis added]".

Although mathematically gifted children are characterised by the quality of their reasoning (Johnson, 1983), these children require appropriate and challenging learning experiences to facilitate their cognitive development (Henningsen & Stein, 1997; Hoeflinger, 1998). Whilst "learning to count large numbers" may only appear to be a small step in the development of mathematical power, learning how to generate knowledge is a crucial milestone in the development of a creative mathematician.

REFERENCES

Clement, Rod. (1990). *Counting on Frank.* Sydney: Angus & Robertson.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences.* London: Heinemann.

Hoeflinger, M. (1998). Developing mathematically promising students. *Roeper Review, 20*(4), 244-247.

Johnson, M. L. (1983). Identifying and teaching mathematically gifted elementary school students. *Arithmetic Teacher, 30*(5), 25-26.

Jones, G., Thornton, C., & Putt, I. (1994). A model for nurturing and assessing multidigit number sense among first grade children. *Educational Studies in Mathematics, 27*(2), 117-143.

Kruteskii, V. A. (1976). *The psychology of mathematical abilities in school children.* Chicago: University of Chicago Press.

Lupkowski-Shoplik, A. E., & Assouline, S. G. (1994). Evidence of extreme mathematical precocity: Case studies of talented youths. *Roeper Review, 16*(3), 144-151.

National Association of Teachers of Mathematics (1998). *Principles and standards for school mathematics: Discussion draft.* Reston, VA: National Association of Teachers of Mathematics.

Schwartz, D. (1985). *How much is a million?* New York, NY: Mulberry.

List of Figures

Figure 1

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536
1 (thousand) 536
21 (thousand) 536
721 (thousand) 536
8 (million) 721 (thousand) 536

Figure 2

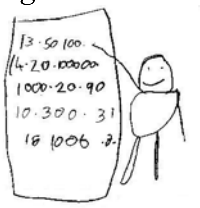

Figure 3

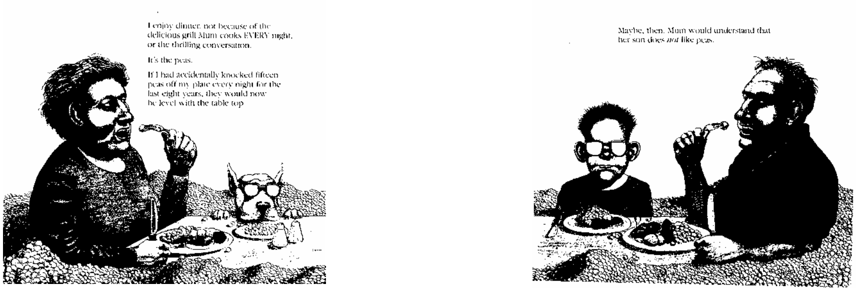

Figure 4

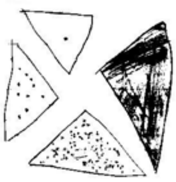

Figure 5

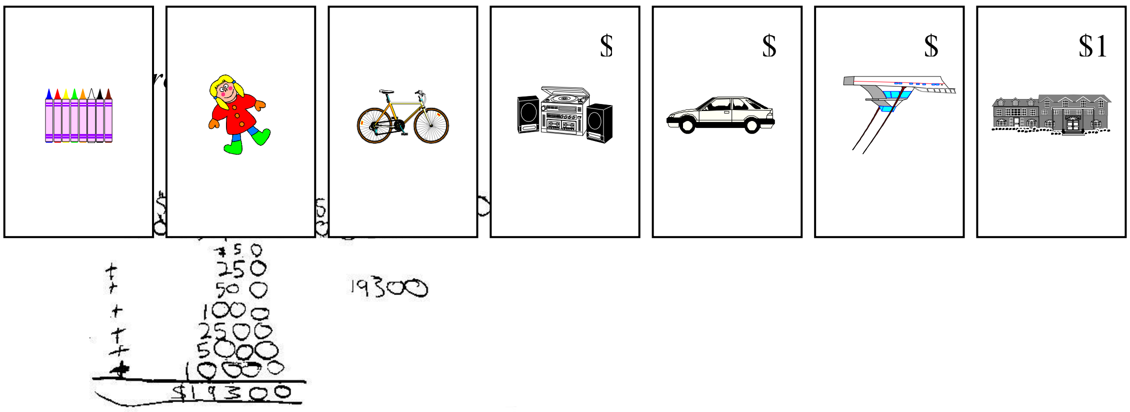

Figure 7